Fourth Korea-Japan Workshop on Algebra and Combinatorics

(sponsored by BK21-CoDiMaRO, PMI, KOSEF, KRF and JSPS)

Date: February 1-2, 2008
Place: Room 404, Mathematical Science Building, POSTECH

Program

February 1 (Friday)

10:00 − 10:40 Jin Ho Kwak (POSTECH)
    Enumeration and genus distribution of maps on surfaces

11:00 − 11:40 Paul Terwilliger (Wisconsin and Kanazawa University)
    A Drinfel'd polynomial for tridiagonal pairs.

12:00 − 15:00 Lunch

15:00 − 15:40 YoungJu Choie (POSTECH)
    Broue-Enguehard maps and Atkin-Lehner involutions

15:50 − 16:30 Masaaki Harada (Yamagata University)
    Two recent results in extremal self-dual codes

16:50 − 17:30 Jong Yoon Hyun (POSTECH)
    Generalized MacWilliams identities and their applications to perfect binary codes

17:40 − 18:20 Sang-il Oum (KAIST)
    Well-quasi-ordering of skew-symmetric matrices of bounded rank-width

February 2 (Saturday)

9:30 – 10:10 Takao Komatsu (Hirosaki University)
Three term recurrence relations and continued fractions

10:25 – 11:05 Hajime Tanaka (Tohoku University)
A bilinear form relating two Leonard pairs

11:20 – 12:00 Rie Hosoya (International Christian University, Tokyo)
The Terwilliger algebra of the Johnson graph with respect to a subset

12:00 – 15:00 Lunch

15:00 – 15:40 Akira Hiraki (Osaka Kyoiku University)
Title: Distance-regular graphs with $c_2 > 1$ and $a_1 = 0 < a_2$

15:50 – 16:30 Jack Koolen (POSTECH)
The Bannai-Ito Conjecture

16:50 – 17:30 Mitsugu Hirasaka (Pusan National University)
Observation for the noncommutative association scheme of order 15

17:40 – 18:20 Edwin van Dam (Tilburg University)
Amorphic association schemes

Abstracts

Jin Ho Kwak (POSTECH)

Title: Enumeration and genus distribution of maps on surfaces

Abstract: Two 2-cell embeddings $\imath : X \to S$ and $\jmath : X \to S$ of a connected graph X into a closed orientable surface S are *congruent* if there are an orientation-preserving surface homeomorphism h on S and a graph automorphism γ of X such that $\imath h = \gamma \jmath$. When we restrict γ as the identity, we say two embeddings are *equivalent*. A 2-cell embedding $\imath : X \to S$ of a graph X into a closed orientable surface S is sometimes described combinatorially by a pair $(X; \rho)$ called a map, where ρ is a product of disjoint cycle permutations each of which is the permutation of the dart set of X initiated at the same vertex following the orientation of S. First we review the enumerating the equivalence or congruence classes of embeddings of a graph and also their genus distribution problem.

The *mirror image* of a map $(X; \rho)$ is the map $(X; \rho^{-1})$, and one of the corresponding embeddings is called the *mirror image* of the other. A 2-cell embedding of X is *reflexible* if it is congruent to its mirror image. Mull et al. [Proc. Amer. Math. Soc. 103(1988) 321-330] developed an approach for enumerating the congruence classes of 2-cell embeddings of graphs into closed orientable surfaces. In this talk we also introduce a method for enumerating the congruence classes of reflexible 2-cell embeddings of graphs into closed orientable surfaces, and apply it to the complete graphs, the bouquets of circles, the dipoles and the wheel graphs to count their congruence classes of reflexible or nonreflexible (called chiral) embeddings.

Paul Terwilliger (Wisconsin and Kanazawa University)

Title: A Drinfel'd polynomial for tridiagonal pairs

Abstract: We introduce a polynomial invariant for tridiagonal pairs called the Drinfel'd polynomial. We discuss how this polynomial generalizes the classical Drinfel'd polynomial associated with the loop algebra for sl_2 or $U_q \mathrm{sl}_2$). We compute the roots of the Drinfel'd polynomial for the case in which the associated tridiagonal pair is a Leonard pair. This is joint work with Tatsuro Ito.

YoungJu Choie (POSTECH)

Title: Broue-Enguehard maps and Atkin-Lehner involutions

Abstract: Let a be one of the ten integers such that the sum of their divisors divide 24. For each such a, (except 15) we give a map from an algebra of polynomial invariants of some finite group to the algebra of modular forms invariant under the Atkin-Lehner group of level a. These maps are motivated and inspired by constructions of modular lattices from self-dual codes over rings. This work generalizes Broue-Enguehard work in level one and three obtained from binary and ternary codes. This is joint work with P.Solé

Masaaki Harada (Yamagata University)

Title: Two recent results in extremal self-dual codes

Abstract: In my talk, I present two recent results on the existence and the classification of extremal self-dual codes.

The existence of binary extremal doubly even self-dual codes was known for the following lengths 8, 16, 24, 32, 40, 48, 56, 64, 80, 88, 104 and 136. The first example of a binary

extremal doubly even self-dual code of length 112 is given. This length is the smallest length for which no extremal doubly even self-dual code of length $n \not\equiv 0 \pmod{24}$ has been constructed.

The second result is about the classification of ternary extremal self-dual codes. All extremal ternary self-dual codes of lengths up to 24 have been classified. We complete the classification of ternary extremal self-dual codes of length 28 using the known classification of 28-dimensional unimodular lattices with minimum norm 3. This is joint work with Akihiro Munemasa and Boris Venkov.

Jong Yoon Hyun (POSTECH)

Title: Generalized MacWilliams identities and their applications to perfect binary codes

Abstract: We present generalized MacWilliams identities for binary codes. These identities naturally lead to the concepts of the local weight distribution of a binary code with respect to a word u and its MacWilliams u-transform. In the case that u is the all-one word, these ones correspond to the weight distribution of a binary code and its MacWilliams transform, respectively. We identify a word v with its support, and consider v as a subset of $\{1, 2, \cdots, n\}$. For two words u, w of length n such that their intersection is the empty set, define the u-face centered at w to be the set $\{z \cup w : z \subseteq u\}$. A connection between our MacWilliams u-transform and the weight distribution of a binary code in the u-face centered at the zero word is presented. As their applications, we also investigate the properties of a perfect binary code. For a perfect binary code C, the main results are as follows: first, it is proved that our local weight distribution of C is uniquely determined by the number of codewords of C in the orthogonal u-face centered at the zero word. Next, we give a direct proof for the known result, concerning the weight distribution of a coset of C in the u-face centered at the zero word, by A.Y. Vasil'eva without using induction. Finally, it is proved that the weight distribution of C in the orthogonal u-face centered at w is uniquely determined by the codewords of C in the u-face centered at the zero word.

Sang-il Oum (KAIST)

Title: Well-quasi-ordering of skew-symmetric matrices of bounded rank-width

Abstract: A set Q is well-quasi-ordered by a quasi-order $\leq$ if every infinite sequence $a_1, a_2, \ldots$ of Q has $i < j$ such that $a_i \leq a_j$. Robertson and Seymour (1990) proved that graphs of bounded tree-width are well-quasi-ordered by the graph minor relation. Geelen, Gerards, and Whittle (2002) proved that matroids representable over a fixed finite field are well-quasi-ordered by the matroid minor relation if they have bounded branch-width. Oum (2004) proved that graphs of bounded rank-width are well-quasi-ordered by the vertex-minor relation by using isotropic systems defined by A. Bouchet in 80's. In this talk, we discuss a common generalization of above three results in terms of skew-symmetric matrices over a fixed finite field of bounded rank-width. A quasi-order of skew-symmetric matrices, called pivot-minor, is defined by using the principal pivot transformation and the principal submatrix. To prove this we extend the notion of isotropic systems to arbitary fields and prove that isotropic systems are essentially equivalence classes of skew-symmetric matrices that are equivalent by the principal pivot transformation.

Takao Komatsu (Hirosaki University)

Title: Three term recurrence relations and continued fractions

Abstract: In this talk we investigate linear three-term recurrence formulae $Z_n = T(n)Z_{n-1} + U(n)Z_{n-2}$ ($n \geq 2$) with sequences of integers $(T(n))_{n\geq 0}$ and $(U(n))_{n\geq 0}$, which are ultimately periodic modulo m, e.g.

$$
\begin{aligned}
(T(n) \bmod m)_{n\geq 0} &= \left(a_0, a_1, a_2, \ldots, a_\rho, \overline{T_1, T_2, \ldots, T_w} \right) , \\
(U(n) \bmod m)_{n\geq 0} &= \left(b_0, b_1, b_2, \ldots, b_\rho, \overline{U_1, U_2, \ldots, U_w} \right) .
\end{aligned}
$$

This is an extension of linear three-term recurrence formulae for $z_n = Z_{rn+i}$ with $0 \leq i < r$, when $(T(n))_{n\geq 0}$ and $(U(n))_{n\geq 0}$ belonging to regular or non-regular Hurwitz-type continued fraction expansions. Using this result we show that the sequence $(Z_n)_{n\geq 0}$ is ultimately periodic modulo m. In addition, we consider a non-regular continued fraction expansion for infinite reciprocal sums of Fibonacci and Lucas numbers. The sequence of rational numbers generated by truncating its expansion also yields interesting three-term recurrence relations.

Hajime Tanaka (Tohoku University)

Title: A bilinear form relating two Leonard pairs

Abstract: Leonard pairs provide a purely linear algebraic framework for the Leonard's theorem, and nowadays the study of Leonard pairs is an active area of research. In this talk, we introduce the notion of a balanced bilinear form that relates two Leonard pairs. We give several characterizations of balanced bilinear forms, and also discuss how they arise in the theory of Q-polynomial distance-regular graphs (i.e., (i) subsets such that width plus dual width equals the diameter; (ii) thin irreducible modules for the Terwilliger algebra).

Rie Hosoya (International Christian University)

Title: The Terwilliger algebra of the Johnson graph with respect to a subset

Abstract: Brouwer, Godsil, Koolen and Martin introduced two parameters w and w^* of a subset C in a Q-polynomial distance-regular graph Γ. They showed that $w + w^* \geq D$, where D denotes the diameter of Γ, and if equality holds, C induces a Q-polynomial distance-regular graph whenever C is connected. For example, a subset C with $w + w^* = D$ in the Johnson graph $J(N, D)$ induces the Johnson graph $J(N - w^*, w)$. In this talk, we determine irreducile modules of the Terwilliger algebra of $J(N, D)$ with respsct to C with $w + w^* = D$. This is a joint work with Hajime Tanaka.

Akira Hiraki (Osaka Kyoiku University)

Title: Distance-regular graphs with $c_2 > 1$ and $a_1 = 0 < a_2$

Abstract : Let Γ be a distance-regular graph of diameter $d \geq 3$ with $c_2 > 1$. Let m be an integer with $1 \leq m \leq d - 1$. We say Γ *satisfies the condition* $(SC)_m$ if for any pair of vertices at distance m in Γ there exists a strongly closed subgraph of diameter m containing them. In this talk we give some necessary and sufficient condition for that Γ satisfies the condition $(SC)_m$. Applying this result we prove that a distance-regular graph with classical parameters (d, b, α, β) such that $c_2 > 1$ and $a_1 = 0 < a_2$ satisfies the condition $(SC)_i$ for $i = 1, 2, \ldots, d-1$. In particular, either $(b, \alpha, \beta) = (-2, -3, -1 - (-2)^d)$ or $(b, \alpha, \beta) = \left(-3, -2, -\frac{1 + (-3)^d}{2} \right)$ holds.

Jack Koolen (POSTECH)

Title: The Bannai-Ito Conjecture

Abstract: In the early 1980's Bannai and Ito conjectured that for a fixed $k \geq 3$, there are finitely many distance-regular graphs with valency k. In a series of papers they showed that this conjecture is true for $k = 3, 4$ and also for the class of bipartite distance-regular graphs. In this talk we discuss some recent developments towards this conjecture. This is joint work with Sejeong Bang and Vincent Moulton.

Mitsugu Hirasaka (Pusan National University)

Title: Observation for the noncommutative association scheme of order 15

Abstract: Let G be a transitive permutation group of Ω with a block Δ and $\Sigma := \{\Delta^x \mid x \in G\}$ where both $|\Delta|$ and $|\Sigma|$ are primes with $gcd(|\Delta|, |\Sigma|-1) = gcd(|\Delta|-1, |\Sigma|) = 1$. In this talk we aim to determine when the permutation character of G on Ω is multiplicity-free, or equivalently, the orbitals under G form a commutative association scheme.

Edwin van Dam (Tilburg University)

Title: Amorphic association schemes

Abstract: An association scheme is called amorphic if any fusion of its relations gives rise to an association scheme. In an amorphic association scheme all relations correspond to strongly regular graphs. More specifically, if there are at least three nontrivial relations, then these strongly regular graphs are of (negative) Latin square type. In this talk we shall give a survey of results on amorphic association schemes, and highlight some related issues.

Organizing Committee:
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